

Density matrix of the superposition of excitation on coherent states with thermal light and its statistical properties

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A beam's density matrix that is described by the superposition of excitation on coherent states with thermal noise (SECST) is presented, and its matrix elements in Fock space are calculated. The maximum information transmitted by the SECST beam is derived. It is more than that by coherent light beam and increases as the excitation photon number increases. In addition, the nonclassicality of density matrix is demonstrated by calculating its Wigner function.

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I. INTRODUCTION

Recently, much attention has been paid to the excitation on coherent states (ECS) [1, 2, 3, 4, 5, 6]. As pointed out in Refs.[2, 3], the single photon ECS causes a classical-to-quantum (nonclassical) transition. The ECSs can be considered as a generalization of coherent states [7, 8] and number eigenstates. All these states can be used as signal beams in optical communications field, in which the nonclassicality of signals plays an important role.

However, in reality, signal beams are usually mixed with thermal noise. Statistical properties of the superposition of (squeezed) coherent states with thermal light (SCST) have been investigated by calculating the photon number matrix elements $\langle N | \rho | M \rangle$ of SCST's density matrix [9, 10]. These properties are useful in quantum optics and quantum electronics (e.g. how lasers working well above threshold, heterodyne detection of light, etc.) [11]. Some general properties of the density matrices which describe coherent, squeezed and number eigenstates in thermal noise are studied in Ref.[12]. It is found that the information transmitted by the superposition of number eigenstates with thermal light (SNET) beam is less than that by the SCST beam [13].

In this paper, we investigate statistical properties of the superposition of ECS with thermal light (SECST). We present the relevant density matrix in Fock space and derive the Mandel Q parameter. The SECST field can exhibit a significant amount of super-Poissonian photon statistics (PPS) due to the presence of thermal noise for excitation photon number $m = 0$; while for $m \neq 0$ the SECST field can present the sub-PPS when the thermal mean photon number is less than a threshold value. In addition, the threshold value increases as m increases. We also calculate the maximum information (channel capacity) transmitted by the SECST beam, which increases as m increases. In addition, the nonclassicality of density matrix is also presented by calculating the Wigner function of the SECST.

Our paper is arranged as follows. In Sec. II we present the density matrix ρ that describes the SECST and calculate its matrix elements in Fock space by using the normal

ordered form of ρ . The PPS distributions are discussed in Sec. III. The maximum information is calculated in Sec. IV. Sec. V is devoted to deriving the Wigner function of the SECST and discussing its nonclassicality in details. Conclusions are summarized in the last section.

II. EXCITATION ON COHERENT STATES WITH THERMAL NOISE

Firstly, let us briefly review the excitation on coherent states (ECSs). The ECSs, first introduced by Agarwal and Tara [1], are the result of successive elementary one-photon excitations of a coherent state, and is an intermediate state in between the Fock state and the coherent state, since it exhibits the sub-Poissonian character. Theoretically, the ECSs can be obtained by repeatedly operating the photon creation operator a^\dagger on a coherent state, so its density operator is

$$\rho_0 = C_{\alpha,m} a^{\dagger m} |\alpha\rangle \langle \alpha| a^m, \quad (1)$$

where $C_{\alpha,m} = [m! L_m(-|\alpha|^2)]^{-1}$ is the normalization factor, $|\alpha\rangle = \exp(-|\alpha|^2/2 + \alpha a^\dagger) |0\rangle$ is the coherent state [7, 8], and $L_m(x)$ is the m th-order Laguerre polynomial.

The SECST is described by the density matrix [12]

$$\rho = \int \frac{d^2 z}{\pi} P(z) D(z) \rho_0 D^\dagger(z), \quad (2)$$

$$P(z) = \frac{1}{\bar{n}_t} \exp \left[-\frac{|z|^2}{\bar{n}_t} \right], \quad (3)$$

where $D(z) = \exp(za^\dagger - z^*a)$ is the displacement operator, and \bar{n}_t is the mean number of thermal photons for $\rho_0 \rightarrow |0\rangle \langle 0|$. We can easily prove that $\text{Tr} \rho = 1$, as it should be. In fact,

$$\begin{aligned} \text{Tr} \rho &= \int \frac{d^2 z}{\pi} P(z) \text{Tr} [D(z) \rho_0 D^\dagger(z)] \\ &= \int \frac{d^2 z}{\pi} P(z) \text{Tr} (\rho_0) \\ &= \int \frac{d^2 z}{\pi} P(z) = 1. \end{aligned} \quad (4)$$

A. Normal ordering form of the SECST

For the simplicity in our later calculation, we first perform the integration in Eq.(2) by using the technique of integration within an ordered product (IWOP) of operators [14, 15]. Using the normal ordering form of the vacuum projector $|0\rangle\langle 0| =: \exp(-a^\dagger a) :$, we can reform Eq.(2) as the following form

$$\begin{aligned} \rho &= C_{\alpha,m} e^{-|\alpha|^2} \int \frac{d^2 z}{\pi} P(z) D(z) : a^{\dagger m} \\ &\quad \times \exp(\alpha a^\dagger + \alpha^* a - a^\dagger a) a^m : D^\dagger(z) \\ &= \frac{C_{\alpha,m}}{\bar{n}_t} : \exp\left(-|\alpha|^2 - a^\dagger a + a^\dagger \alpha + \alpha a^*\right) \\ &\quad \times \int \frac{d^2 z}{\pi} \exp\left[-\frac{1+\bar{n}_t}{\bar{n}_t} |z|^2\right] \\ &\quad \times \exp\left[(a^\dagger - \alpha^*)z + (a - \alpha)z^*\right] \\ &\quad \times (a^\dagger - z^*)^m (a - z)^m : . \end{aligned} \quad (5)$$

In the last step of (5), we noticed that for any operator $f(a^\dagger, a)$

$$D(z) f(a^\dagger, a) D^\dagger(z) = f(a^\dagger - z^*, a - z). \quad (6)$$

Making two independent variable displacements,

$$a^\dagger - z^* \rightarrow \beta^*, a - z \rightarrow \beta,$$

(note that operators a^\dagger, a can be considered as C-number within the normal order $: :$), thus Eq.(5) can be rewritten as

$$\begin{aligned} \rho &= \frac{C_{\alpha,m}}{\bar{n}_t} : \exp\left(-|\alpha|^2 - \frac{1}{\bar{n}_t} a^\dagger a\right) \\ &\quad \times \int \frac{d^2 \beta}{\pi} \beta^{*m} \beta^m \exp\left[-\lambda_t^{-2} |\beta|^2\right. \\ &\quad \left.+ \left(\alpha^* + \frac{a^\dagger}{\bar{n}_t}\right)\beta + \left(\frac{a}{\bar{n}_t} + \alpha\right)\beta^*\right] : \\ &= \frac{C_{\alpha,m}}{\bar{n}_t} \lambda_t^{2m+2} : \exp\left(-|\alpha|^2 - \frac{1}{\bar{n}_t} a^\dagger a\right) \\ &\quad \times \int \frac{d^2 \beta}{\pi} \beta^{*m} \beta^m \exp\left[-|\beta|^2 + A^\dagger \beta + A \beta^*\right] : , \end{aligned} \quad (7)$$

where we have set $\lambda_t = \sqrt{\bar{n}_t/(1+\bar{n}_t)}$ and $A = \lambda_t(\frac{1}{\bar{n}_t}a + \alpha)$. Then using the integration expression of two-variable Hermite polynomial $H_{m,n}$ [16],

$$\begin{aligned} &(-1)^n e^{-\xi\eta} H_{m,n}(\xi, \eta) \\ &= \int \frac{d^2 z}{\pi} z^n z^{*m} \exp\left[-|z|^2 + \xi z - \eta z^*\right], \end{aligned} \quad (8)$$

we can put Eq.(7) into

$$\begin{aligned} \rho &= \frac{C_{\alpha,m}}{\bar{n}_t} \lambda_t^{2m+2} : (-1)^m H_{m,m}(A^\dagger, -A) \\ &\quad \times \exp\left[-\frac{(a-\alpha)(a^\dagger - \alpha^*)}{\bar{n}_t + 1}\right] : . \end{aligned} \quad (9)$$

In particular, when $m = 0$, corresponding to the case of superposition of coherent state with thermal noise, Eq.(9) reduces to

$$\rho = \frac{1}{\bar{n}_t + 1} D(\alpha) : e^{-\frac{a^\dagger a}{\bar{n}_t + 1}} : D^\dagger(\alpha), \quad (10)$$

which can be directly checked by using Eqs.(2) and (3) as well as noticing $\rho_0 = |\alpha\rangle\langle\alpha|$.

Further employing the relation between Hermite polynomial and Laguerre polynomial [16],

$$H_{m,n}(\xi, \kappa) = \begin{cases} n! (-1)^n \xi^{m-n} L_n^{m-n}(\xi\kappa), & m > n \\ m! (-1)^m \kappa^{n-m} L_m^{n-m}(\xi\kappa), & m < n \end{cases}, \quad (11)$$

we can see that

$$\begin{aligned} \rho &= \frac{1}{L_m(-|\alpha|^2)} \frac{\bar{n}_t^m}{(1+\bar{n}_t)^{m+1}} : L_m(-A^\dagger A) \\ &\quad \times \exp\left[-\frac{(a-\alpha)(a^\dagger - \alpha^*)}{\bar{n}_t + 1}\right] : . \end{aligned} \quad (12)$$

Eqs.(9) and (12) are the normal ordering form of the SECST. From these it is convenient to calculate the phase space distributions, such as Q-function, P-representation and Wigner function.

B. The matrix elements $\langle N | \rho | M \rangle$

Now we calculate the matrix elements of ρ in Eq.(2) between two number states $\langle N |$ and $| M \rangle$, i.e., $\langle N | \rho | M \rangle$. Employing the overcompleteness of coherent states, one can express the matrix elements $\langle N | \rho | M \rangle$ as

$$\langle N | \rho | M \rangle = \int \frac{d^2 \beta d^2 \gamma}{\pi^2} \langle N | \beta \rangle \langle \beta | \rho | \gamma \rangle \langle \gamma | M \rangle, \quad (13)$$

where the overlap between the coherent state and the number state is given by

$$\langle \gamma | M \rangle = \frac{1}{\sqrt{M!}} e^{-|\gamma|^2/2} \gamma^{*M}, \quad (14)$$

and the matrix elements $\langle \beta | \rho | \gamma \rangle$ can be obtained from Eq.(9) due to ρ 's normal ordering form,

$$\begin{aligned} \langle \beta | \rho | \gamma \rangle &= (-1)^m \frac{C_{\alpha,m}}{\bar{n}_t} \lambda_t^{2m+2} e^{-|\alpha|^2/(\bar{n}_t+1)} \\ &\quad \times \frac{\partial^{2m}}{\partial \tau^m \partial \tau'^m} \exp[-\tau\tau' + \lambda_t \tau \alpha^* - \lambda_t \alpha \tau'] \\ &\quad \times \exp\left\{\left(\frac{\alpha + \bar{n}_t \gamma}{\bar{n}_t + 1} + \frac{\lambda_t \tau}{\bar{n}_t}\right) \beta^* - \frac{1}{2} |\beta|^2\right. \\ &\quad \left.- \frac{1}{2} |\gamma|^2 + \left(\frac{\alpha^*}{\bar{n}_t + 1} - \frac{\lambda_t \tau'}{\bar{n}_t}\right) \gamma\right\}_{\tau=\tau'=0}, \end{aligned} \quad (15)$$

where we have used the generating function of two-variable Hermite polynomial $H_{m,n}$,

$$H_{m,n}(x, y) = \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp[-tt' + tx + t'y] \Big|_{t=t'=0}. \quad (16)$$

When $M = N$, $\langle N | \rho | N \rangle$ is just the photon number distribution of the SECST. Then combining with Eqs.(15), (13) and (14), after a lengthy but straightforward calculation, one can get the matrix elements $\langle N | \rho | M \rangle$, (without loss of generality, let $M \geq N$)

$$\langle N | \rho | M \rangle = \frac{(-1)^N}{\sqrt{M!N!}} \frac{\lambda_t^{2N} C_{\alpha,m}}{\bar{n}_t + 1} e^{-|\alpha|^2} \frac{\partial^{2m}}{\partial v^m \partial v'^m} \cdot \left\{ e^{\lambda_t^2 v v'} H_{M,N} \left(\frac{v'}{\bar{n}_t + 1}, -\frac{v}{\bar{n}_t} \right) \right\}_{v=\alpha, v'=\alpha^*}, \quad (17)$$

where we have used the integral formula [17]

$$\int \frac{d^2\beta}{\pi} f(\beta^*) \exp \left\{ -|\beta|^2 + \tau\beta \right\} = f(\tau), \quad (18)$$

and another expression of two-variable Hermite polynomial $H_{m,n}$,

$$H_{m,n}(\xi, \kappa) = \sum_{l=0}^{\min(m,n)} \frac{m!n!(-1)^l \xi^{m-l} \kappa^{n-l}}{l!(n-l)!(m-l)!}. \quad (19)$$

In particular, when $m = 0$, noticing $M \geq N$ and Eq.(11), Eq.(17) reduces to

$$\langle N | \rho | M \rangle = \sqrt{\frac{N!}{M!}} \alpha^{*M-N} \frac{(\bar{n}_t)^N}{(\bar{n}_t + 1)^{M+1}} \times e^{-|\alpha|^2/(\bar{n}_t+1)} L_N^{M-N} \left[-\frac{|\alpha|^2}{\bar{n}_t(\bar{n}_t + 1)} \right], \quad (20)$$

which is just the Glauber-Lachs formula [9] when $\bar{n}_t = (e^{\beta\omega} - 1)^{-1}$. While for $\alpha = 0$, corresponding to the case of superposition of number state with thermal light, using Eq.(19), Eq.(17) becomes

$$\langle N | \rho | M \rangle = \delta_{M,N} P_N, \quad (21)$$

where $(k_0 = \max[0, m - N])$

$$P_N = \frac{m!N!}{\bar{n}_t + 1} \sum_{k=k_0}^m \frac{1}{k!} \frac{\left(\frac{\bar{n}_t}{\bar{n}_t + 1} \right)^{k+N} [\bar{n}_t(\bar{n}_t + 1)]^{k-m}}{(k + N - m)! [(m - k)!]^2}. \quad (22)$$

Eq.(21) is just the result of Ref. [13].

III. SUB-POISSONIAN PHOTON STATISTICS

To see clearly the photon statistics properties of the SECST, in this section, we pay our attention to the variance of the photon number operator $\langle (\Delta \hat{n})^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$. In particular, we will examine the evolution of the Mandel Q parameter defined as

$$Q = \frac{\langle (a^\dagger a)^2 \rangle}{\langle a^\dagger a \rangle} - \langle a^\dagger a \rangle = \frac{\langle a^2 a^{\dagger 2} \rangle - \langle a a^\dagger \rangle^2 - \langle a a^\dagger \rangle}{\langle a a^\dagger \rangle - 1}, \quad (23)$$

which measures the derivation of the variance of the photon number distribution of the field state under consideration from the Poissonian distribution of the coherent state. $Q = 1, Q > 1$ and $Q < 1$ correspond to Poissonian photon statistics (PPS), super-PPS and sub-PPS, respectively.

In order to calculate the average value in Eq.(23), we first calculate the value of $\langle \alpha | a^n a^{\dagger m} | \alpha \rangle$. In fact, using

$$\langle \alpha | a^{m+n} a^{\dagger m+n} | \alpha \rangle = (m+n)! L_{m+n}(-|\alpha|^2) \quad (24)$$

and

$$\int \frac{d^2 z}{\pi} z^n z^{*m} P(z) = \bar{n}_t^m m! \delta_{m,n}, \quad (25)$$

we can evaluate (for writing's convenience, let L_m denote $L_m(-|\alpha|^2)$)

$$\langle a^\dagger a \rangle = \frac{1+m}{L_m} L_{m+1} + \bar{n}_t - 1, \quad (26)$$

and

$$\langle a^2 a^{\dagger 2} \rangle = 2\bar{n}_t^2 + \frac{m+1}{L_m} [4\bar{n}_t L_{m+1} + (m+2) L_{m+2}]. \quad (27)$$

Substituting Eqs.(26) and (27) into (23) leads to

$$Q = \frac{\bar{n}_t(\bar{n}_t - 1) L_m + (2\bar{n}_t - 1)(m+1) L_{m+1}}{(1+m) L_{m+1} + (\bar{n}_t - 1) L_m} + \frac{(m+1)(m+2) L_{m+2} - \frac{(m+1)^2}{L_m} L_{m+1}^2}{(1+m) L_{m+1} + (\bar{n}_t - 1) L_m}. \quad (28)$$

At the zero-temperature limit ($\bar{n}_t \rightarrow 0$), Eq.(28) just reduces to Eq.(2.20) in Ref.[1].

In Fig.1, we display the parameter $Q(n_t, |\alpha|)$ as a function of $(n_t, |\alpha|)$ for different values m . From Fig.1, we see that, for the excitation photon number $m = 0$ (see Fig.1 (a)), $Q(\bar{n}_t = 0, |\alpha|) = 1$ corresponding to coherent state (a PPS); while $Q(\bar{n}_t \neq 0, |\alpha|) > 1$, i.e., the SECST field exhibits a significant amount of super-PPS due to the presence of \bar{n}_t . From Fig.1 (b) and (c), we see that, when $m \neq 0$, the SECST field presents the sub-PPS when \bar{n}_t is less than a threshold value for a given $|\alpha|$; the threshold value increases as m increases. For example, when $|\alpha| = 0$, the threshold values are about 0.414 and 0.481, respectively, for $m = 1$ and $m = 6$.

IV. INFORMATION TRANSMITTED BY THE SECST BEAM

According to the negentropy principle of Brillouin [18], the maximum information I transmitted by a beam is

$$I = S_{\max} - S_{act}, \quad (29)$$

in which S_{\max} and S_{act} represent the maximum entropy and the actual entropy, respectively, possessed by the

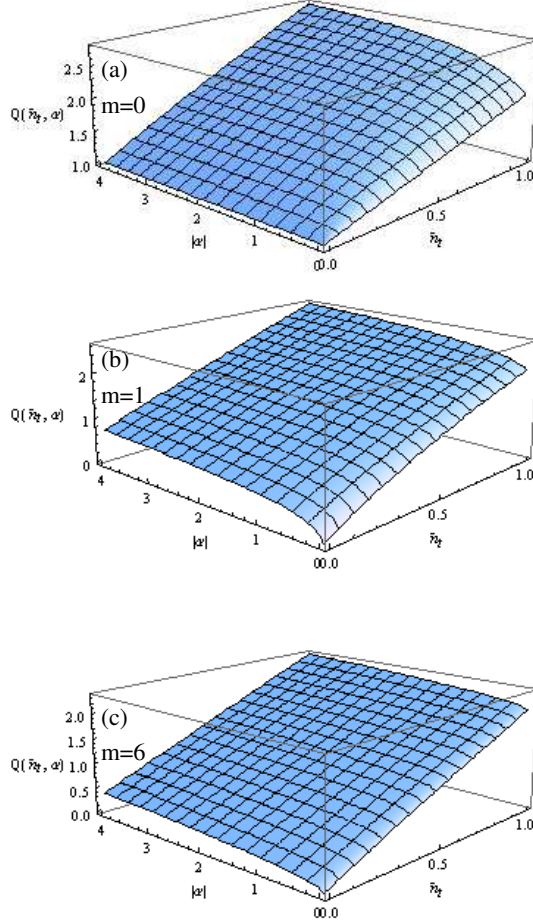


FIG. 1: (Color online) The evolution of Mandel Q parameter as a function of $(\bar{n}_t, |\alpha|)$ for different values m .

quantum mechanical system described by a density matrix ρ . Here the maximum information I is an ideal one transmitted through an ideal optical communication system.

For the SECST system, the actual entropy is

$$S_{act} = -\text{Tr}(\rho \ln \rho) = -\sum_N \sigma_N \ln \sigma_N, \quad (30)$$

where $\rho = \sum_N \sigma_N |N\rangle \langle N|$, and $\sigma_N = \langle N | \rho | N \rangle$. σ_N can be obtained from Eq.(17), i.e.,

$$\sigma_N = \frac{\bar{n}_t^N e^{-|\alpha|^2} C_{\alpha, m}}{(\bar{n}_t + 1)^{N+1}} \frac{\partial^{2m}}{\partial v^m \partial v'^m} \times \left\{ e^{\lambda_t^2 v v'} L_N \left(\frac{-v v'}{\bar{n}_t (\bar{n}_t + 1)} \right) \right\}_{v=\alpha, v'=\alpha^*}, \quad (31)$$

which is independent of the phase of α . On the other hand, for a system in thermal equilibrium, described by the density matrix ρ_{th} , with mean photons number \bar{n}_t ,

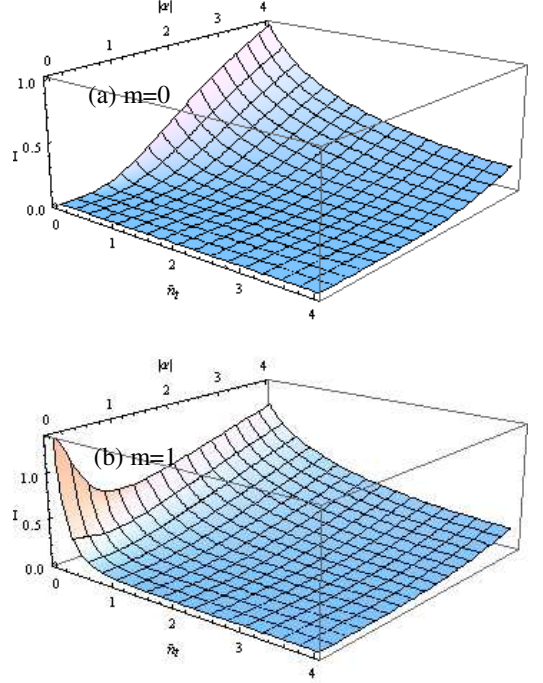


FIG. 2: (Color online) The maximum information $I(\bar{n}_t, |\alpha|)$ as a function of $(\bar{n}_t, |\alpha|)$ for some different values (a) $m = 0$, (b) $m = 1$, (truncating the infinite sum at $N_{\max} = 70$).

its entropy is

$$S = -\sum_N P_N \ln P_N = \ln(1 + \bar{n}_t) + \bar{n}_t \ln \frac{\bar{n}_t + 1}{\bar{n}_t}, \quad (32)$$

where $P_N = \bar{n}_t^N / (\bar{n}_t + 1)^{N+1}$ obtained from Eq.(20) under the condition $m = 0, \alpha = 0$. Note that the maximum entropy of the system is equal to the entropy of a system in thermal equilibrium, with an equal mean number of photons. The mean photons number of the SECST is given by Eq.(26). Therefore, using Eq.(32), we have

$$S_{\max} = \ln \left((1 + m) \frac{L_{m+1}}{L_m} + \bar{n}_t \right) + \left((1 + m) \frac{L_{m+1}}{L_m} + \bar{n}_t - 1 \right) \times \ln \left(\frac{(1 + m) L_{m+1} + \bar{n}_t L_m}{(1 + m) L_{m+1} + (\bar{n}_t - 1) L_m} \right). \quad (33)$$

From Eqs.(29), (30) and (33), we can calculate the maximum information transmitted by the SECST beam. In Fig. 2, the maximum information $I(\bar{n}_t, |\alpha|)$ is plotted as a function of $(\bar{n}_t, |\alpha|)$ for some different values m (truncating the infinite sum at $N_{\max} = 70$). From Fig.2, we can see that, for a given \bar{n}_t , $I(\bar{n}_t, |\alpha|)$ increases as the value $|\alpha|$ increases; for given $|\alpha|$, in general, $I(\bar{n}_t, |\alpha|)$ grows up as the value \bar{n}_t increases. In order to see clearly the effect of different parameter m to $I(\bar{n}_t, |\alpha|)$, we presented a plot in Fig.3, from which it is obvious

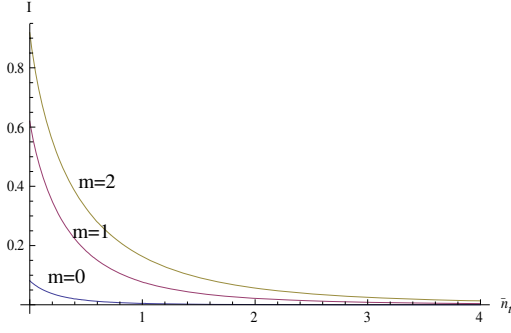


FIG. 3: (Color online) The maximum information $I(\bar{n}_t, |\alpha|=1)$ as a function of (\bar{n}_t) for some different values (a) $m=0$, (b) $m=1$, (c) $m=2$ (truncating the infinite sum at $N_{\max}=70$).

that $I(\bar{n}_t, |\alpha|)$ becomes bigger due to the presence of m , and increases as m increases. In other words, the maximum information transmitted by the SECST beam is larger than that by the SCST ($m=0$). The channel of ECS can carry with more information than that of coherent state. In Ref. [13], Vourdas has pointed out that the coherent signals (of known phase) can transmit more information than the number eigenvectors signals. Thus among these three beams, the SECST beam can transmit most information.

V. THE WIGNER FUNCTION OF THE SECST

A. The Wigner function

The Wigner function (WF) plays an important role in quantum optics, especially the WF can be reconstructed from measurements [19, 20]. The WF is a powerful tool to investigate the nonclassicality of optical fields. The presence of negativity in the WF of optical field is a signature of its nonclassicality is often used to describe the decoherence of quantum states. In this section, using the normally ordered form of the SECST, we evaluate its WF. For a single-mode system, the WF is given by [21]

$$W(\gamma, \gamma^*) = \frac{e^{2|\gamma|^2}}{\pi} \int \frac{d^2\beta}{\pi} \langle -\beta | \rho | \beta \rangle e^{2(\beta^* \gamma - \beta \gamma^*)}, \quad (34)$$

where $|\beta\rangle$ is the coherent state and $\gamma = x + iy$. From Eq.(34) it is easy to see that once the normal ordered form of ρ is known, we can conveniently obtain the WF of ρ .

On substituting Eq.(9) into Eq.(34) we obtain the WF

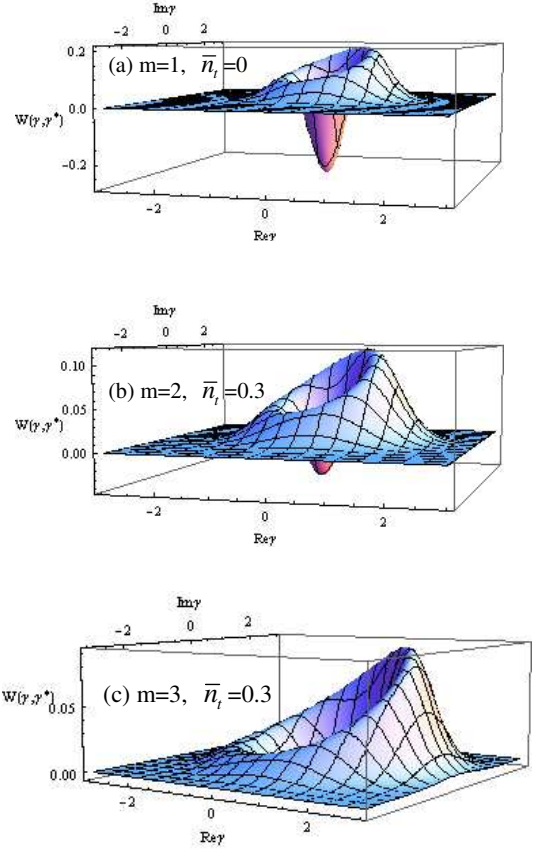


FIG. 4: (Color online) The evolution of the Wigner function of the SECST with $\alpha = 0.2 + 0.2i$ for several different values m and \bar{n}_t .

of the SECST

$$\begin{aligned} W(\gamma, \gamma^*) &= \frac{(\lambda_t^2 A_1^2)^m C_{\alpha, m}}{\pi (2\bar{n}_t + 1)} \exp \left\{ -\frac{2|\alpha - \gamma|^2}{2n_t + 1} \right\} \\ &\times (-1)^m H_{m, m} \left(\frac{A_2^*}{A_1}, -\frac{A_2}{A_1} \right) \\ &= \frac{(\lambda_t^2 A_1^2)^m \exp \left\{ -\frac{2|\alpha - \gamma|^2}{2\bar{n}_t + 1} \right\}}{\pi (2\bar{n}_t + 1) L_m(-|\alpha|^2)} L_m \left(-|A_2|^2 / A_1^2 \right), \end{aligned} \quad (35)$$

where we have set

$$\begin{aligned} A_1^2 &= 1 - \frac{1}{(2\bar{n}_t + 1)\bar{n}_t}, \\ A_2 &= \frac{\lambda_t (\bar{n}_t + 1)}{(2\bar{n}_t + 1)\bar{n}_t} (2\bar{n}_t \alpha - \alpha + 2\gamma). \end{aligned} \quad (36)$$

Noticing that $L_m(-|\alpha|^2) > 0$, and $L_m[-|A_2|^2 / A_1^2] > 0$ when $1 - \frac{1}{(2\bar{n}_t + 1)\bar{n}_t} > 0$, thus the WF of the SECST is always positive under the condition of $\bar{n}_t > 1/2$. In

particular, when $m = 0$, Eq.(35) becomes

$$W(\gamma, \gamma^*) = \frac{1}{\pi(2\bar{n}_t + 1)} \exp\left\{-\frac{2|\alpha - \gamma|^2}{2\bar{n}_t + 1}\right\}, \quad (37)$$

which corresponds to the thermal state with mean photon number \bar{n}_t . While for $\alpha = 0$, $A_2 \rightarrow 2\gamma\lambda_t(\bar{n}_t + 1)/[(2\bar{n}_t + 1)\bar{n}_t]$, $|A_2|^2/A_1^2 \rightarrow 4|\gamma|^2(\bar{n}_t + 1)/\{(2\bar{n}_t + 1)[(2\bar{n}_t + 1)\bar{n}_t - 1]\} \equiv \xi$, Eq.(35) yields

$$W(\gamma, \gamma^*) = \frac{[(2\bar{n}_t + 1)\bar{n}_t - 1]^m}{\pi(2\bar{n}_t + 1)^{m+1}(\bar{n}_t + 1)^m} e^{-\frac{2|\gamma|^2}{2\bar{n}_t + 1}} L_m(-\xi). \quad (38)$$

At the zero-temperature limit, $T \rightarrow 0$, $\bar{n}_t \rightarrow 0$, Eq.(37) reduces now into $\frac{1}{\pi} \exp(-2|\alpha - \gamma|^2)$, i.e., the WF of coherent state (a Gaussian form), which can be seen from Eq.(2) yielding $\rho = |\alpha\rangle\langle\alpha|$ under the condition $m = 0$; while Eq.(38) becomes $\frac{1}{\pi}(-1)^m e^{-2|\gamma|^2} L_m(4|\gamma|^2)$, corresponding to the WF of number state.

Using Eq.(35), the WFs of the SECST are depicted in Fig.4 in phase space with $\alpha = 0.2 + 0.2i$ for several different values m and \bar{n}_t . It is easy to see that the negative region of WF gradually disappears as m and \bar{n}_t increases.

B. The marginal distributions of the SECST

We now find the probability distribution of position or momentum—the marginal distributions, by performing the WF either over the variable y or the variable x , respectively. Using Eqs.(35) and (36) we can derive (denote $\gamma = x + iy$, $\alpha = q + ip$)

$$\begin{aligned} P(x, \bar{n}_t) &\equiv \int W(x, y) dy \\ &= \frac{(\lambda_t^2 A_1^2)^m C_{\alpha, m} [m!]^2 e^{-\frac{2(q-x)^2}{2\bar{n}_t + 1}}}{\sqrt{2\pi(2\bar{n}_t + 1)} (2\bar{n}_t - 1)^m} \\ &\times \sum_{k=0}^m \frac{2^{2k-m} \bar{n}_t^k}{k! [(m-k)!]^2} |H_{m-k}(E_1)|^2, \end{aligned} \quad (39)$$

where $H_m(x)$ is single variable Hermite polynomial and $E_1 = [(2\bar{n}_t - 1)\alpha + 2x + 2ip]/\sqrt{2(2\bar{n}_t + 1)}$. Eq.(39) is the marginal distribution of WF of the SECST in “ x -direction”.

On the other hand, performing the integration over dx yields the other marginal distribution in “ y -direction”,

$$\begin{aligned} P(y, \bar{n}_t) &= \frac{(\lambda_t^2 A_1^2)^m C_{\alpha, m} [m!]^2 e^{-\frac{2(p-y)^2}{2\bar{n}_t + 1}}}{\sqrt{2\pi(2\bar{n}_t + 1)} (2\bar{n}_t - 1)^m} \\ &\times \sum_{k=0}^m \frac{2^{2k-m} \bar{n}_t^k}{k! [(m-k)!]^2} |H_{m-k}(E_2)|^2, \end{aligned} \quad (40)$$

where $E_2 = i(2\bar{n}_t\alpha - \alpha + 2q + 2iy)/[\sqrt{2(2\bar{n}_t + 1)}]$. As expected, the two marginal distributions are both real.

VI. CONCLUSIONS

In summary, we have investigated the photon statistics properties of the SECST, described by the density matrix $\rho(2)$. We have calculated the matrix elements $\langle N | \rho | M \rangle$ in Fock space and the Mandel Q parameter. It is found that the SECST field exhibits a significant amount of super-PPS due to the presence of thermal noise (\bar{n}_t) for excitation photon number $m = 0$ and that, for $m \neq 0$ and a given $|\alpha|$, the SECST field presents the sub-PPS when \bar{n}_t is less than a threshold value. In addition, the threshold value increases as m increases. We have presented the maximum information (channel capacity) transmitted by the SECST beam. It is shown that the maximum information transmitted increases as m increases. This implies that among the coherent signals, the eigen-number signals and the ECS in thermal light, the last one can transmit the most information. Further, as one of the photon statistical properties, the Wigner function and the marginal distributions of the SECST have also been derived, from which one can clearly see the nonclassicality. The negative region has no chance to be present when the average photon number \bar{n}_t of thermal noise exceeds $1/2$. The marginal distributions are related to the Hermite polynomial.

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- [1] G. S. Agarwal, K. Tara, *Phys. Rev. A* **43**, 492 (1991).
 - [2] A. Zavatta, S. Viciani, M. Bellini, *Science* **306**, 660 (2004).
 - [3] A. Zavatta, S. Viciani, M. Bellini, *Phys. Rev. A* **72**, (2006) 023820.
 - [4] Y. Li, H. Jing and M. S. Zhan, arXIV:

- quantum-ph/0610143v1.
- [5] D. Kalamidas, C. C. Gerry, A. Benmoussa, *Phys. Lett. A* **372**, 1937-1940 (2008).
- [6] Truong Minh Duc, Jaewoo Noh, *Opt. Commun.* **281**, 2842 (2008).
- [7] R. J. Glauber, *Phys. Rev.* **130**, 2529 (1963); R. J.

- Glauber, *Phys. Rev.* **131**, 2766 (1963).
- [8] J. R. Klauder and B. S. Skargerstam, Coherent States, (World Scientific, Singapore, 1985).
 - [9] B. R. Mollow and R. J. Glauber, *Phys. Rev.* **160**, 1076 (1967); G. Lachs, *Phys. Rev. B* **138**, 1012 (1965).
 - [10] A. Vourdas, *Phys. Rev. A* **34**, 3466 (1986).
 - [11] B. Saleh, Photoelectrons Statistics (Springer-Verlag, Berlin, 1978).
 - [12] A. Vourdas, *Phys. Rev. A* **39**, 206 (1989).
 - [13] A. Vourdas, *Phys. Rev. A* **37**, 3890 (1988).
 - [14] H.-Y. Fan, H. R. Zai and J. R. Klauder, *Phys. Rev. D* **35**, 1831 (1987); H.-Y. Fan, H.-L. Lu and Y. Fan, *Ann. Phys.* **321**, 480 (2006).
 - [15] A. Wünsche, *J. Opt. B: Quantum Semiclass. Opt.* **1**, R11 (1999).
 - [16] A. Wünsche, *J. Phys. A: Math. and Gen.* **33**, 1603 (2000).
 - [17] R. R. Puri, Mathematical Methods of Quantum Optics, (Springer-Verlag, Berlin, 2001), Appendix A.
 - [18] L. Brillouin, *J. Appl. Phys.* **24**, 1152 (1953); A. Wehrl, *Rev. Mod. Phys.* **50**, 221 (1978).
 - [19] K. Vogel and H. Risken, *Phys. Rev. A* **40**, 2847 (1989).
 - [20] D. T. Smithy et al., *Phys. Rev. Lett.* **70**, 1244 (1993).
 - [21] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge: Cambridge University Press, 1997).